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Assessment of flatness error by regression analysis

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A R T I C L E I N F O

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A B S T R A C T

The geometric tolerancing (GPS or GD&T) has increasing importance in machine design, manufacturing and measuring. The geometric tolerances define the deviation of the different kinds of geometric elements in more sophisticated way, than dimensional tolerances. Nevertheless, the application of them requires more wariness in design process, manufacturing process planning and measuring. The key element is the measuring, when the requirements have to be inspected according to the standards, and striving to the maximal productivity and accuracy. The paper presents a new method to evaluate the flatness deviation by limited number of measured points and it uses regression analysis based on partial point sets. The method considers the observation, than the number of measured points increases the evaluated flatness values. The article presents the new method, the results of test, which support it and the result of verification tests also.

1. Introduction

A machine part has several properties, which are defined based on the purpose and working properties and expectation. Beside the shape, size and material, the tolerances are the most important properties. The tolerancing is a very complex design task, where lot of circumstances have to be considered, not only the design purposes, but the manufacturing, assembly, measuring, maintenance and cost.

The tolerancing activity covers the macro and micro geometry. In case of macro geometry, the dimensional tolerances and the geometric tolerances can be applied. The dimensional tolerance means the tolerance of distances, diameters, radius etc. The geometric tolerances mean the possible error of geometric elements, like line, circle, cylinder, and plane. The geometric tolerances are defined by standards [1,2]. The application of geometric tolerances has several aspects. The first is the notation in the shop drawing, the second is the functional justification, the third is the manufacturing aspect and the fourth is the measuring aspect. The standards describe the first level only.

Tolerances generate requirements for the manufacturing and measuring process, and through the parameters of manufacturing methods the different kind of errors are defined and controlled. Sheth and George [3] presents the effect of the process parameters on the flatness in case of face milling. Nowakowski et al. [4] investigates the slot milling technology, and present the effect of the milling parameters and milling strategies on the flatness, parallelism and perpendicularity. The productivity is assessed too. Wang and So [5] presents the effect of the process parameters of grinding to the micro (surface roughness) and micro (flatness) parameters. The ball-burnishing can evolve the micro accuracy of the surface too. Based on the result of Kovács et al. [6] the path parameter of the process modifies not only the surface roughness and the micro hardness, but the flatness error too.

The result of the manufacturing can be controlled by measuring. The measuring of geometric error has several parameters, which has influence to the measured value. The first step of the evaluation process is to measure the coordinates of the points of the surface. Kawalec and Magdziak [7] found, then the higher number of measured points improves the measured geometric error in case of free form surfaces. Lakota and Görög [8] presents the effect of number of measured points on the measured flatness error. A unified scanning method was used, and it was found, that not only the number of points increase the flatness, but the scanning direction has importance too. Moulai-Khatir et al. [9] presents the effect of the investigated points and the type of evaluation of flatness.

The type of the measuring device (layout, design, size, accuracy) has an important role too. It can be a coordinate measuring machine, a measuring arm, or different kind of scanning devices. In case of contact method, the type of the probe is an important feature. It can be a touch probe or a scanning head. The circumstances of the coordinate measurement have influence on the results, as Štribac et al. [10] presents. The position of the workpiece, the temperature and the size of the tip have the largest effects on the roundness. The environment (temperature, vibration etc.) has effect on the result of the measuring process too. Moroni and Petro [11] presents the process of inspection planning in a wide content, and several aspects of coordinate measurement are

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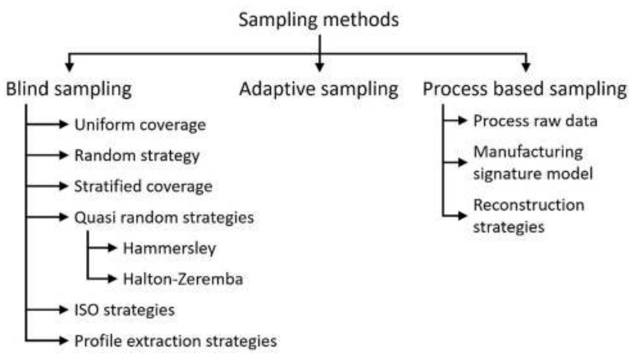

Fig. 1. Classification of point sampling methods (based on [12]).

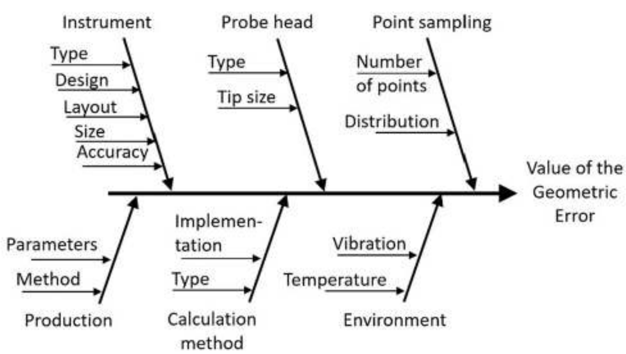

Fig. 2. Influence parameters of the geometric errors.

described, like the point sampling strategies, probe configuration and the path of the probe.

The next character of the process is the point sampling, which defines the number of points and their distribution. The number of measured points and the point sampling strategy is an important part of the inspection planning [11]. There are three types of point sampling methods [12]. In case of (1) blind sampling strategies the sampling pattern is defined before the measuring, and does not change from part to part. The uniform coverage, the random strategy, the stratified coverage, the quasi random strategies (like Hammersley or Halton-Zeremba), ISO strategies and different kind of profile extraction methods form this group. The (2) adaptive sampling strategy is not a predefined strategy. Based on a starting sample set, considering criterions the adaptive method chooses the next sampling point till the stop condition. The (3) process based sampling methods consider manufacturing information in order to define the best point pattern (Fig. 1).

The calculation method has effect on the geometric error value. Based on measured points the form and position tolerances can be evaluated, but several mathematical methods and their implementations can be used. Beside the white-box methods, black-box methods can be used too, like genetic algorithm or different search algorithms [13,14]. The most often used white-box methods are the following:

- Least square (LS) method, when the regression geometry is defined based on points by minimizing the distance of the points from the regression geometry.
- Minimum zone (MZ) method, when the position and orientation of the two parallel investigation elements is optimized by minimizing the distance between the two objects.
- Envelope method (EM), when a cover geometric feature is located to 3 points, and every other points there are under (or inside) the feature. The distance of the farthest point is the geometric error. During the evaluation, a cover geometric element has to be found where this distance is the smallest.

The Fig. 2 summarizes the parameters in form of Ishikawa diagram, which have influence on the geometric errors. During the manufacturing and measuring process planning these parameters have to be considered.

The flatness error describes the deviation of a plane surface from the theoretical plane. The flatness tolerance defines the permissible level of this error. The flatness error is the distance of two parallel planes, which limit the produced flat surface (Fig. 3) [1]. The two parallel planes have 3 deg of freedom (DOF), one linear in vertical direction and two angular DOFs in the horizontal plane. During the calculation of flatness error, the position of the parallel planes should be defined.

Based on the definition of the standard, the flatness error can be calculated as the maximum distance between a plane and points of the produced surface. If one point of a plane is $\underline{P_0} = [P_{0x}; P_{0y}; P_{0z}]$ and the normal vector is $\underline{N} = [N_x; N_y; N_z]$ the distance of any point, which is described by $\underline{P_i} = [P_{ix}; P_{iy}; P_{iz}]$, is

$$D_i = \frac{N_x\cdot(P_{0x} - P_{ix}) + N_y\cdot(P_{0y} - P_{iy}) + N_z\cdot(P_{0z} - P_{iz})}{\sqrt{N_x^2 + N_y^2 + N_z^2}} \quad (1)$$

In case of LS method, the regression plane must be determined by $\underline{P_0}$ and $\underline{N}$, when the sum of square of distances from the measured points have minimal value. The flatness error is the sum of the distance from the plane of the upper and lower furthest points. During the implementation of the algorithm, the problem is, how can be the best position of the plane found in general state, when the number of points is unknown (or can be arbitrary).

In case of MZ method the regression plane is determined by $\underline{N}$, the $\underline{P_0}$ is not important, it can be [0,0,0]. The right position of the plane is, when the distance between the closest and the furthest points from the plane is minimal. During the implementation a fast and robust search algorithm required in order to find the optimal position of the normal vector.

The envelope method covers the measured point cloud. The plane must be fit to three points of the point cloud, and any other points must be below or above to this plane. It means two different solutions. The flatness error is the distance of the furthest point from the plane. The task is to find the three points, which ensure the smallest distance. From the viewpoint of the implementation this is the most complicated method. In case of high number of points there are several variations for selecting three points and evaluating the location of other points and calculating the distances.

Based on the literature and the previous tests, the number of points has important effect on the calculated value of the flatness error. If the

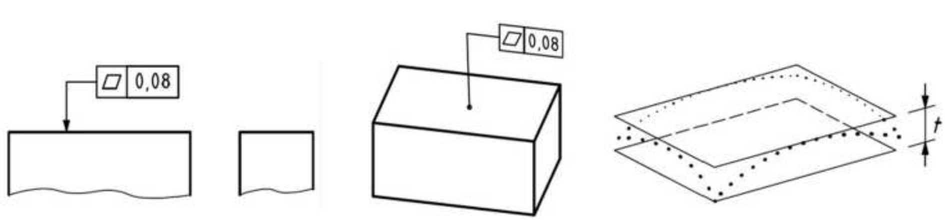

Fig. 3. Definition of the flatness (ISO 1101).

Table 1
Machining conditions of test samples.

	Sf#1	Sf#2	Sf#3	Sf#4	Sf#5	Sf#6
Method	Face milling		Face turning		Face milling	
Strategy	Zig-Zag		–		Zig-Zag	Spiral
Machine tool	UF-231	MAZAK A410-II	E400-1000		MAZAK A410-II	
Type	Manual	CNC	Manual		CNC	
D_c [mm]	80	50	–		63	
z [-]	7	4	1		6	
v_c [m/min]	60		(100)		180	
n [1/min]	240	382	190		910	
f; f_z [mm]	0.046		0.6	0.2	0.09	
v_f [mm/min]	78	70	115	40	490	
a_p [mm]	1		0.5		1	
a_e [mm]	40	25	–		31.5	

D_c – cutting tool diameter; z – number of teeth; v_c – cutting speed; n – spindle speed.
f, f_z – feed, feed per tooth; v_f – feed speed; a_p – depth of cut; a_e – width of cut.

Table 2
Reference values of flatness in mm.

	Sf#1	*Sf#2*	*Sf#3*	*Sf#4*	*Sf#5*	*Sf#6*
FL_Ref [mm]	*0.0343*	*0.0127*	*0.0427*	*0.0572*	*0.0124*	*0.0204*

number of sample points increases, the value of the flatness increases too [12], so more accurate values are achieved, but the measuring and the calculation process becomes time consuming. During the design of measuring process our aim is to decrease the uncertainty of the measuring and increase the accuracy. If only 3 points are measured on the surface, the flatness error is 0. If more points are selected, the selected points give better representation of the machined surface; the point cloud contains more information. So the value of the flatness error becomes larger, approaches to the real value. The "real" flatness error can be calculated based on point cloud, where the distance between neighbours goes to zero.

The aim of the current research is to increase the accuracy of random or quasi random point sampling methods and reduce the measuring time. In the current article, the effect of the number of points is presented, and a new, regression based estimation model is introduced.

The next chapter presents the method, materials and equipment of the test related to the flat surface machining and flatness measuring. Then the results of the measuring are described. The fourth chapter presents the new concept of the assessment of flatness deviation based on regression analysis and the verification by blind tests. The conclusion closes the paper.

2. Methods and equipment

The algorithm was tested on six machined flat surfaces of the same dimensions 175 mm × 155 mm. The test surfaces were machined using different technologies, methods and machine tools (Table 1). The test parts were made 42CrMo4 (1.7225) pre-hardened low alloyed steel (Rm = 860–1060 MPa).

Surfaces #1 and #2 were machined in a conventional and CNC milling machine by zig-zag technology, with the same cutting speed and feed per tooth. Surfaces #3 and #4 were machined by face turning with the same parameters, except the feed. As for #5 and #6 surfaces, face milling technology was applied with the same tool and parameters, and with different tool path strategies.

The coordinate values of the investigated flat surfaces were measured by means of Mitutoyo Crysta-Plus 544 coordinate measuring machine. The measurement was performed in a discrete sampling mode with a contact probe (tip diameter is 3 mm). Sampling was carried out in *34 × 30 = 1020* uniformly distributed points on the examined surface. The reference values of the flatness error were calculated by Kotem SurfaceProfile v5. The reference values can be seen in the Table 2.

During the investigation a minimum zone method was implemented [15], where a random hill climbing algorithm was applied for the best plane searching.

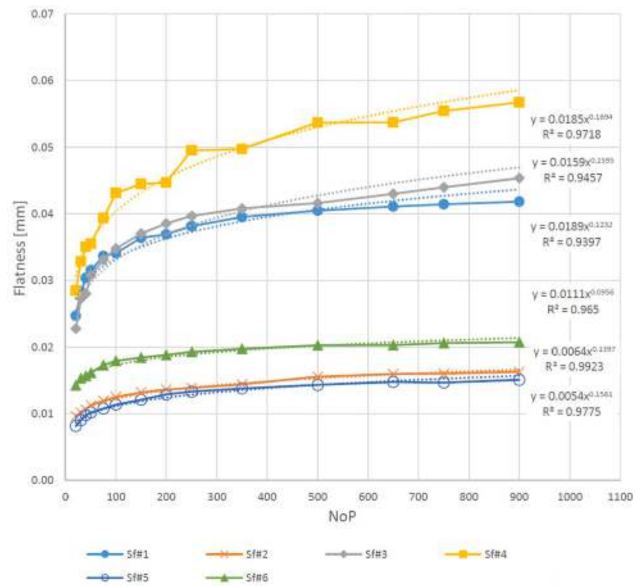

Fig. 4. Flatness in case of random point sampling in function of number of points.

Table 3
Values of coefficients of regression NoP = 20–900.

SF#	*A*	*B*	*R^2adj*
1	*0.0189*	*0.1232*	*0.9397*
2	*0.0064*	*0.1397*	*0.9923*
3	*0.0159*	*0.1595*	*0.9457*
4	*0.0185*	*0.1694*	*0.9718*
5	*0.0054*	*0.1561*	*0.9775*
6	*0.0111*	*0.0956*	*0.9923*

3. Results

The effect of the number of point was investigated through the random point method. Different number of points were selected from the measured 1020 points, and the values of flatness were calculated in case of the 6 specimens. The process was repeated 100 times and the average values were analysed. The number of points were 20, 30, 40, 50, 75, 100, 125, 150, 200, 250, 350, 500, 650, 750 and 900.

Although, the dynamic of the changing is different during the various test parts, but the continuous growing of the values can be observed (Fig. 4). This result is consistent with the literature [7,12,16]. The changing of the flatness in function of number of investigated points can be described by power function in the next form:

$$FL = A \cdot NoP^{B} \quad (2)$$

where

FL: flatness,
NoP: number of points,
A, B: coefficients.

In case of 6 surfaces, the values of the coefficients can be seen in Table 3 (MS Excel). The quality of the regression is very good, the R^2adj parameters are larger than 0.9. Unfortunately, the measuring process of 1020 points takes about 45 min.

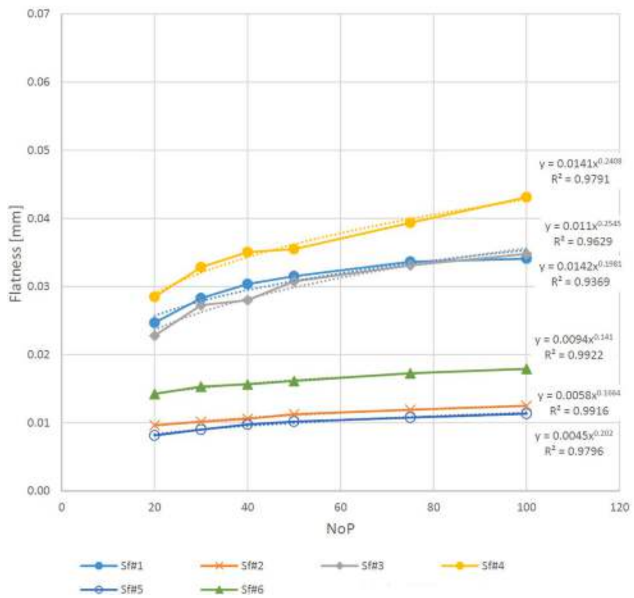

Fig. 5. Flatness values in function of number of measured points.

Table 4
Values of coefficients of regression NoP = 20–100.

SF#	*A*	*B*	R^2adj
1	*0.0142*	*0.1981*	*0.9369*
2	*0.0058*	*0.1664*	*0.9916*
3	*0.0110*	*0.2545*	*0.9629*
4	*0.0141*	*0.2408*	*0.9791*
5	*0.0045*	*0.2020*	*0.9796*
6	*0.0094*	*0.1410*	*0.9922*

If only 100 points are chosen, the measuring time can be reduced, but the measured values go far from reference values. Partial sets of points are investigated (*NoP = 20; 30; 40; 50; 75; 100*), and found, then the trend and regression are very similar (Fig. 5; Table 4). If the relationship between the regression of full and the partial sets can be found, the real flatness value can be estimated based on limited sets of points. Unfortunately, the random point sampling method has a large uncertainty. The random points cannot cover the surface with appropriate density, so another method should be used for point sampling.

4. Regression method of the assessment of flatness

The similarity in regression is the base of the proposed method. The method consists of the following steps:

- Measure the points' coordinates.
- Divide the measured points into six sets.
- Assess the flatness error to the 1st set, then the 1st and the 2nd and so on, and finally to all points.
- Calculate the coefficients of the regression model (Eq. (2)) by least square method.
- Estimate the flatness by the (1) equation with *NoP = 1020*.

The first question is how the limited sets of points can be selected. The random point sampling method has a large problem, it cannot cover the whole surface with same density in every case and the results of repeated measure with different points have a large deviation. The uniform coverage is not better, because in case of periodical macro and micro surface structure it can make false results. So the solution is the application of quasi-random sampling methods [12]. The Halton-Zeremba method was chosen, based on the literature overview. In case of Halton-Zeremba method the relative coordinates of the point can be defined as following [12,16]:

$$x_i = \frac{i}{NoP} \tag{3}$$

$$y_j = \sum_{j=0}^{k-1} b_{ij}' \cdot 2^{(-j-1)} \tag{4}$$

where

i: the number of the points (*0* to *(NoP-1)*)
b_{ij}: the j^{th} bit of the binary representation of *i*
b_{ij}': the transformed value of b_{ij}
$b_{ij}^{'} = b_{ij}$, if *j* is even,
$b_{ij}^{'} = 1 - b_{ij}$, if *j* is odd,

In the implementation 64 points were determined (*NoP = 64*) in order to decrease the measuring time, so *i* is between *0* and *63*. The binary representation of *i = 63* and the b_{ij}' are:

j	*5*	*4*	*3*	*2*	*1*	*0*
b_{ij}	*1*	*1*	*1*	*1*	*1*	*1*
b_{ij}'	*0*	*1*	*0*	*1*	*0*	*1*

Based on Halton-Zeremba method 64 points were defined (HZ64). The subdivision of point set considers the coverage of all surface. The Table 5 and Fig. 6 shows the 6 sets of points for flatness calculation.

In the 1st subset there are 16 points. The 2nd to 5th subsets contain 9 points and the 6th subset contains 12 points. The selection of the points from Halton-Zeremba 64 points set was arbitrary, and only the equable covering was considered. The different way of division can be used also.

Based on the six calculated values of the flatness, the *A* and *B* coefficients of regression model can be determined based on least square method. In case of least square method the sum of squares of differences have to be minimized through the model parameters. The sum of squares (SS) is:

$$SS = \sum_{k=1}^{6} \left[FL_k - \left(A \cdot NoP_k^B \right) \right]^2 \tag{5}$$

where

k: number of point sets,
FL_k: calculated flatness in case of k^{th} calculation,
NoP_k: number of points in case of k^{th} calculation.
SS has minimum value, if

Table 5
Point sets.

k	*Point ID (i)*																
1	*0*	*1*	*2*	*3*	*19*	*20*	*21*	*22*	*38*	*39*	*40*	*41*	*57*	*58*	*59*	*60*
2	*7*	*8*	*9*	*26*	*27*	*28*	*45*	*46*	*47*							
3	*13*	*14*	*15*	*32*	*33*	*34*	*51*	*52*	*53*							
4	*16*	*17*	*18*	*35*	*36*	*37*	*54*	*55*	*56*							
5	*10*	*11*	*12*	*29*	*30*	*31*	*48*	*49*	*50*							
6	*4*	*5*	*6*	*23*	*24*	*25*	*42*	*43*	*44*	*61*	*62*	*63*				

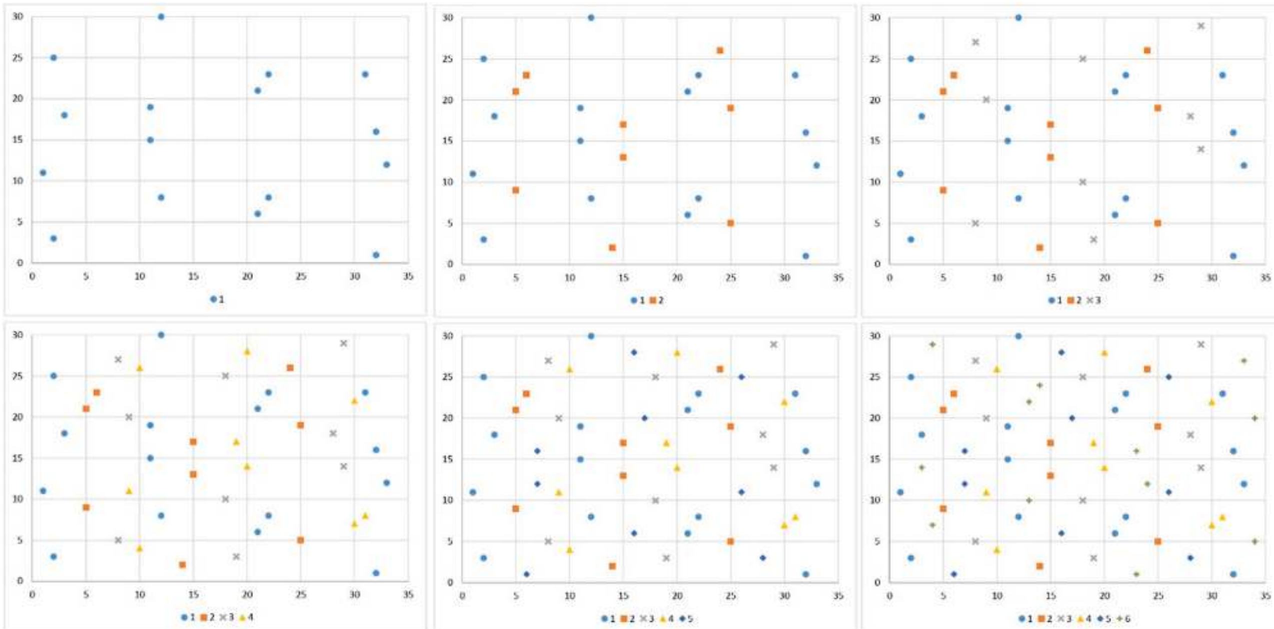

Fig. 6. Point sets for calculation of flatness.

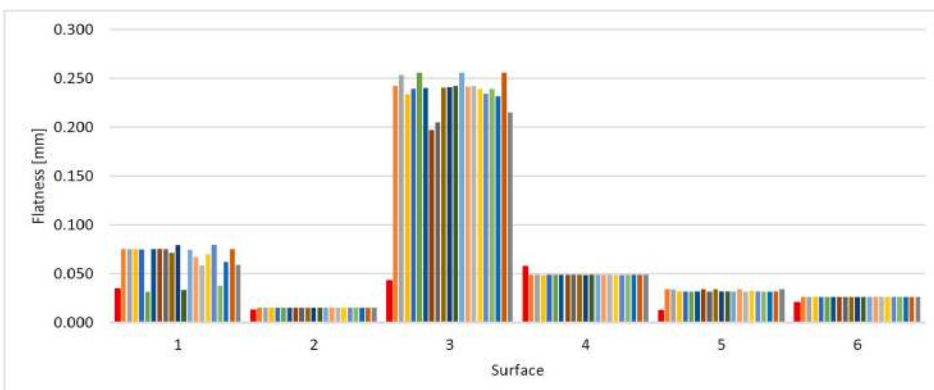

Fig. 7. Results of test runs on 6 data sets.

$$\frac{dSS}{dA} = 0 \text{and} \frac{dSS}{dB} = 0. \tag{6}$$

After the calculation

$$B = \frac{6 \cdot \sum_{k=1}^{6} (lgNoP_k \cdot lgFL_k) - \sum_{k=1}^{6} (lgNoP_k) \cdot \sum_{k=1}^{6} (lgFL_k)}{6 \cdot \sum_{k=1}^{6} (lgNoP_k)^2 - \left[\sum_{k=1}^{6} (lgNoP_k)\right]^2} \tag{7}$$

$$lgA = \frac{B \cdot \sum_{k=1}^{6} (lgNoP_k) - \sum_{k=1}^{6} (lgLF_k)}{-6} \tag{8}$$

$$A = 10^{lgA} \tag{9}$$

The Fig. 7 shows the result of the presented method in case of 20 runs. The calculated values of the flatness are different because of the random hill climbing method. If other method is used to evaluate the flatness error, the differences can be smaller or can be eliminated. The first (red) column shows the reference value. Comparing with the first columns, we can state, that the evaluation method is not as good, as we expected. In case of the 1st and the 3rd surfaces the differences are very large. The cause of this can be found in the nature of the character of the surfaces and in the inaccuracy of the proposed extrapolation theory.

In order to improve the accuracy of the presented method, an additional step should be inserted. The calculated *A* and *B* coefficients must be modified by a modification factor:

$$A' = C_A \cdot A \tag{10}$$

$$B' = C_B \cdot B \tag{11}$$

where

A, B: the original coefficients of the power regression equation,
A', B': the modified coefficients,
C_A, C_B: modification factors.

Based on Table 3 and 4, in case of the whole and partial sets of points the ratio of *A* and *B* coefficients are in narrow ranges (C_A: 1.18–1.42 and C_B: 0.62–0.84), so this simple modification method seems to be successful.

The modification factors were determined based on 6 × 100 = 600 runs by MS Excel Solver optimization module, where the sum of squares of errors was minimized. The values of the factors are $C_A = 1.802$ and $C_B = 0.466$. These coefficients ensure the smallest difference between the modified and the reference flatness values.

The results of the test run with the modified assessment equation can

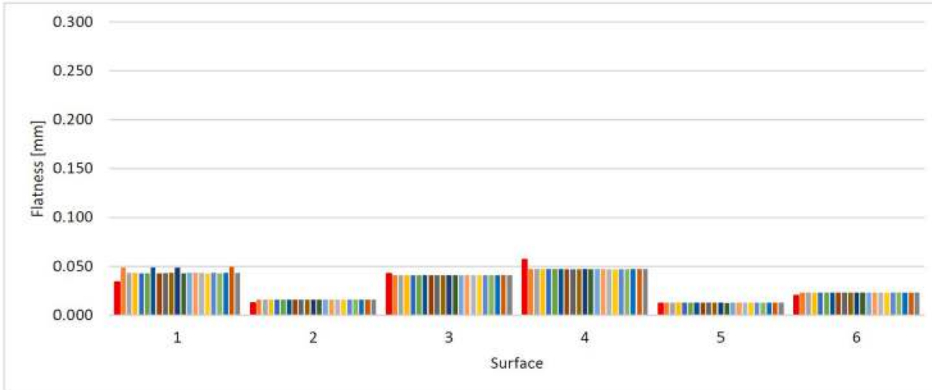

Fig. 8. Results of test runs in case of modified method.

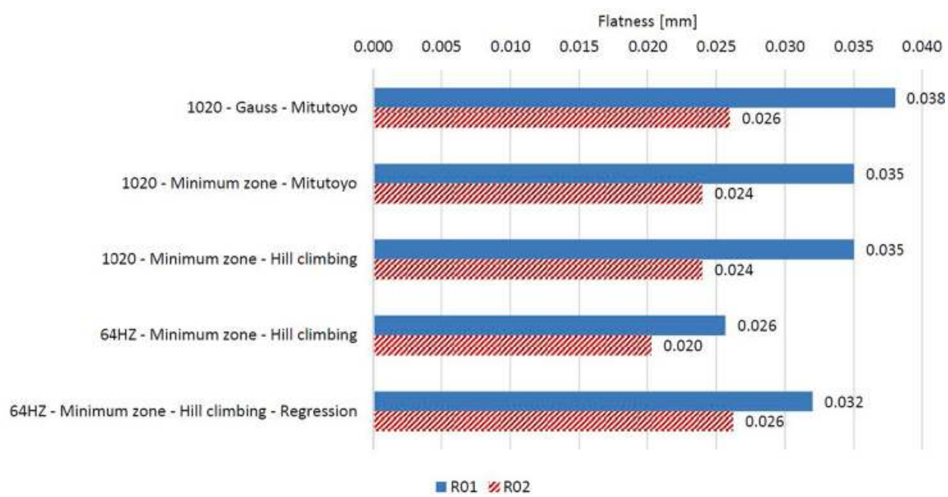

Fig. 9. Values of flatness error in verification.

be seen on the Fig. 8. The evaluated flatness values are closer to the reference values; the accuracy of the measuring process becomes more acceptable. Comparing with the reference values, the largest difference is 16 µm in case of the investigated $6 \times 20 = 120$ test runs. The largest differences are at test surface 1. The effect of the modified parameters is very spectacular in case of surface 1 and 3, where the differences were very large and the repeatability of the runs was poor. Now the repeated runs result almost the same flatness values for each surfaces.

In order to verify the above presented theory, additional two milled surfaces were analysed, #R01 and #R02. The same size flat surfaces were machined by a conventional milling machine with a $D_c = 125$ mm milling head.

Based on 1020 measured point the flatness error of R01 is 0.038 mm by Gauss method of the CMM software and 0.035 mm by minimum zone method of the CMM software. The result of the random hill climbing algorithm is 0.035 mm (Fig. 9). The flatness error of R02 is 0.026 mm by Gauss method of the CMM software and 0.024 mm by minimum zone method of the CMM software. The result of the random hill climbing algorithm is 0.024 mm. The different results demonstrate the effect of the flatness evaluation methods.

Based on 64 Halton-Zeremba (64HZ) points the flatness error is 0.026 mm and 0.020 mm by random hill climbing algorithm, which are smaller than the reference values (0.035/0.024 mm). But, if the presented regression method is applied, the estimated flatness errors are 0.032 mm and 0.026, which are closer to the reference values. It can be concluded, that the calculated flatness error is improved by the regression method. The improvement means, that the calculated value gets closer to the reference value.

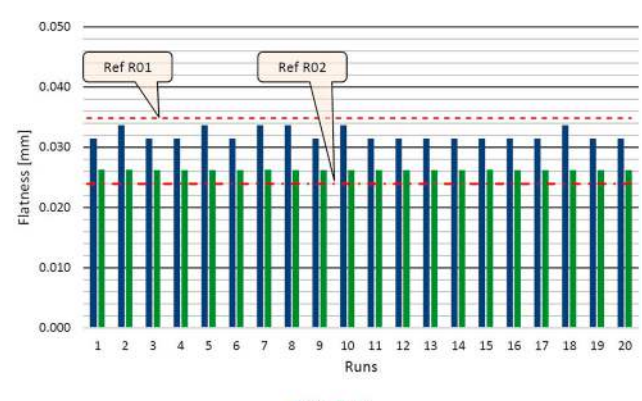

Fig. 10. Results of 20 runs of random hill climbing algorithm in case of verification.

The diagram on Fig. 10 shows the calculated values of 20 repeated runs. The results of 20 runs are little bit different, which is the nature of the random hill climbing algorithm, but the differences are less than 4 and 2 µm from the reference values.

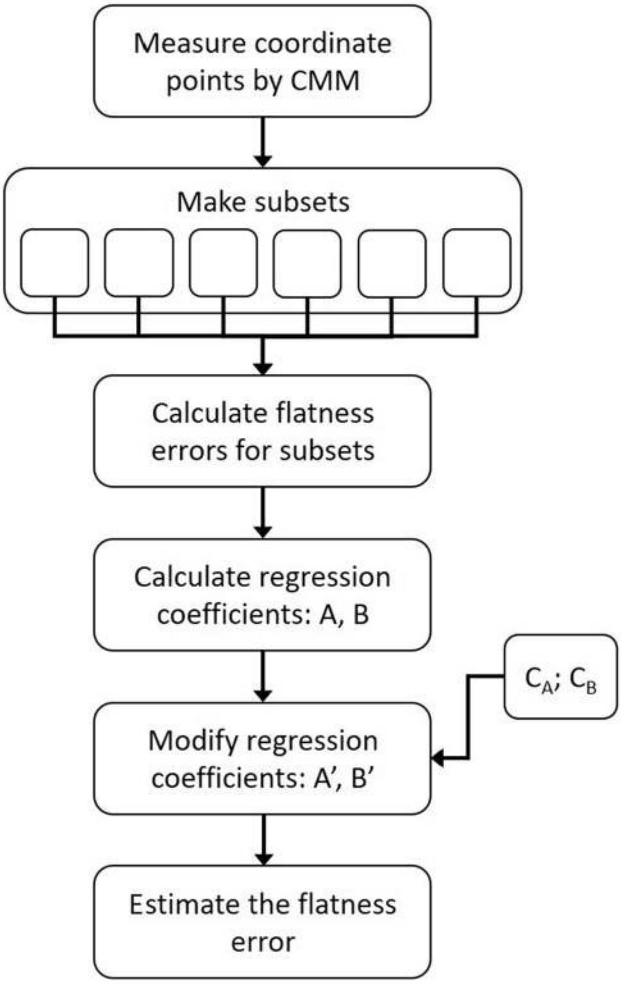

Fig. 11. The process of the assessment of flatness error.

5. Conclusion

The flatness error describes one type of the geometric deviation of a plane surface. The flatness error, as every geometric error, can be investigated from several aspects, like definition, interpretation, measuring, and manufacturing.

In case of measuring the productivity (measuring time) and the accuracy compete with each other. In order to reduce measuring time, a limited number of points should be measured, but the accuracy increases, when more points are used to evaluation of geometric error. The suggested procedure helps to find the equilibrium between productivity and accuracy of the flatness measure. The changing of the flatness in function of number of investigated points can be described by power function. The power function is able to improve the calculated flatness value.

The equable covering of the investigated surface can be ensured by quasi random point sampling methods. In the current research the Halton-Zeremba method is suggested with 64 points. After the measuring process the evaluation consist of the next steps (Fig. 11):

- Define the flatness error to six subsets, where every subset is part of the next.
- Generate the regression curve as power function to these six values.
- Modify the coefficients of the regression model based on previous tests.
- Use extrapolation in order to evaluate the real value of the flatness error. During the extrapolation, the theoretical number of points have to be considered.

In the article the application of this regression method is presented, and demonstrated through independent examples. The suggested method ensures fast measuring process thanks to the small number of measured points, and decreases the error of the result thanks to the extrapolation method.

In case of general use of the method, the definition of "real" flatness error, and the reference density of points, and the size of the surface are important questions, which need more investigation.

In the future result some details can be investigated, like the generation of subsets, the size of the investigated plane or the effect of the nature of the machined surfaces. The further aim is to interpret the method in case of other type of geometric tolerances, like circularity or roundness.

CRediT authorship contribution statement

Balázs Mikó: Conceptualization, Methodology, Validation, Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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